Seat No. ________________

HAM-003-1163004

M. Sc. (Sem. III) (CBCS) Examination

June – 2023

Mathematics : CMT-3004

(Discrete Mathematics)

Faculty Code : 003
Subject Code : 1163004

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) There are five questions.

(2) Answer all the questions.

(3) Each question carries 14 marks.

1 Answer any **seven** of the following : **7×2=14**

(1) Define semigroup. Prove or disprove : $\mathbb{Z}$ with usual subtraction is semigroup.

(2) Define with example : Symmetric closure of R.

(3) Define with example : POSET.

(4) Draw hasse diagram of $(P(A), \subseteq)$ where $A = \{1, 2, 3\}$.

(5) Write down absorption properties for lattice.

(6) Define with example : Complemented lattice.

(7) Define : Atoms and Co-atoms.

(8) Define : Phrase structure grammar and language of a phrase structure grammar.

(9) Define with example : Statement.

(10) Explain negation of a proposition.

2 Answer any **two** from the following questions : **2×7=14**

(a) Let R_1 and R_2 be equivalence relations defined on a nonempty set A then show that, $(R_1 \cup R_2)^{\infty}$ is the smallest equivalence relation on A containing $R_1 \cup R_2$.

(b) State and prove, fundamental theorem of homomorphism of semi groups.

(c) Explain Warshall's algorithm.

3 Answer following **two** : **2×7=14**

(a) Prove that, if $(L, \leq)$ be any finite boolean algebra then there exists a nonempty set A such that $(L, \leq)$ is isomorphic to $(P(A), \subseteq)$.

(b) Define sublattice. Give an example to show that sublattice of a complete lattice need not be complete.

OR

3 Answer following two : **2×7=14**

(a) Prove that, $(L, \leq)$ is a modular lattice if and only if any sub lattice of $(L, \leq)$ is not isomorphic to the pentagon lattice.

(b) Let $G = (V, S, v_0, \mapsto)$ be a phrase structure grammar in which $V = \{v_0, w, a, b, c\}, S = \{a, b, c\}$ and v_0 is the starting symbol for substitutions and the production relation $\mapsto$ is given by

(1) $v_0 \mapsto aw$

(2) $w \mapsto bbw$

(3) $w \mapsto c$. Find $L(G)$.

4 Answer following **two** : **2×7=14**

(a) Explain tautology. Verify by truth table that given compound statements are tautology or not.

(1) $\sim a \vee (a \vee b)$

(2) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

(3) $(\sim a \wedge b) \to b$

(b) Explain logical equivalence. Verify the following by truth table.

(1) $(p \to q) \equiv (\sim p \vee q)$

(2) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(3) $a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

5 Answer any **two** from the following questions : **2×7=14**

(a) Explain the following.

(1) Conjunction of propositions

(2) Disjunction of propositions

(3) Conditional statement

(4) Converse of a conditional statement

(b) State and prove, pumping lemma.

(c) Let $(P, \leq)$ be a POSET. Prove that $(P, \leq^{-1})$ is also a POSET. Where $\leq^{-1}$ is defined by $a \leq^{-1} b$ if $b \leq a, \forall a, b \in P$.

(d) Let $f(x) = x^2 - x - 2$ and R is a relation on $(\mathbb{Z}, +)$ defined by ${}_aR_b$ iff $f(a) = f(b)$. Prove that, R is an equivalence relation on $\mathbb{Z}$ but it is not a congruence relation on $\mathbb{Z}$.

———————